Part II: Demand Side Analysis

Chapter 2

DEMAND THEORY

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Outline

- Market demand function
- Elasticity:
  - Midpoint elasticity
  - Point elasticity
- Own-price elasticity of demand
- Cross-price elasticity of demand
- Income elasticity of demand
- Advertising elasticity of demand

Market Demand Function

- **The market demand function for a product is derived from the <u>horizontal summation of individual direct demand functions</u>.**
- For example, <u>there are 3 individual (or consumer) **direct** demand functions</u> in the market:
  - Consumer 1: $q_1 = f(P)$.
  - Consumer 2: $q_2 = f(P)$.
  - Consumer 3: $q_3 = f(P)$.
- A direct market demand function, Q_D:

$$Q_D = q_1 + q_2 + q_3. \quad (1)$$

Example 1

- Suppose that the total market demand for a product consists of the demands from consumer 1 and consumer 2.
- The demand equations of the two consumers are given as follows:

$$q_1 = f(P) = 20 - 2P$$

$$q_2 = f(P) = 40 - 5P$$

What is the market demand equation for this product?

- **The direct market demand equation is**

$$Q_D = q_1 + q_2 = (20+40) – (2+5)P = 60 – 7P.$$

Example 2

- Suppose that the total market demand for a product consists of three consumer demands.
- The inverse demand equations of three consumers are given by the following equations:
  - **Consumer 1: $P = 10 – 4q_1$.**
  - **Consumer 2: $P = 5 – 2q_2$.**
  - **Consumer 3: $P = 2 – q_3$.**

Question: What is the market demand equation for this product?

Answer: $Q_D = 7 – 1.75P$.

Market Demand Function

The relationship between the quantity demanded (Q_X) and all variables that determine demand. **$Q_X = f$ (Price of good X, Non-price determinants)**

The estimated direct market demand function:

$$\mathbf{Q_X = b_0 + b_1 P_X^{\alpha} + b_2 I^{\beta} + b_3 P_Y^{\gamma} + b_4 P_E^{\delta} + b_5 A^{\theta} + b_6 \, Pop^{\kappa}} \qquad (2)$$

where b_0, b_1, b_2, b_3, b_4, b_5 and b_6 are coefficient parameters; Q_X = quantity demanded for good X; P_X = price of good X; I = consumers' income; P_Y = price of good Y; P_E = future expected price; A = Advertising; and Pop = population.

($\alpha, \beta, \gamma, \delta, \theta, \kappa$) = Exponents of determinants of market demand.

(P_X, I, P_Y, P_E, A, Pop) = Independent variables = Determinants of Q_X.

Coefficient Parameters

b_1, b_2, b_3, b_4, b_5 , b_5, b_6 **=**

Estimated of the change in Q_X with respect to a one-unit change in determinant of demand.

Marginal value of Q_X with respect to determinant of demand (see Math Review).

Coefficient Parameters and Calculus

A direct market demand function:

$$Q_X = b_0 + b_1 P_X^{\alpha} + b_2 I^{\beta} + b_3 P_Y^{\gamma} + b_4 P_E^{\delta} + b_5 A^{\theta} + b_6\ Pop^{\kappa}$$

Coefficient Parameter	Marginal value of Q_X with respect to	Partial derivative of Q_X with respect to the Determinant	
b_1	P_X (holding the other factors constant)	$b_1 = \frac{\partial Q_X}{\partial P_X} < 0$	
b_2	I (holding the other factors constant)	$b_2 = \frac{\partial Q_X}{\partial I} > 0$ $b_2 = \frac{\partial Q_X}{\partial I} < 0$	
b_3	P_Y (holding the other factors constant)	$b_3 = \frac{\partial Q_X}{\partial P_Y} > 0$ $b_3 = \frac{\partial Q_X}{\partial P_Y} < 0$	
b_4	P_E (holding the other factors constant)	$b_4 = \frac{\partial Q_X}{\partial P_E} > 0$	
b_5	A (holding the other factors constant)	$b_5 = \frac{\partial Q_X}{\partial A} > 0$	
b_6	POP (holding the other factors constant)	$b_6 = \frac{\partial Q_X}{\partial POP} > 0$	

Example 3

The market demand for gasoline ($\mathbf{Q_G^D}$), measured in millions of gallons, is given by

$$\mathbf{Q_G^D = 10 - 2P_G + 0.1I + 4P_E},$$

where $\mathbf{P_G}$ is the current price of gasoline per gallon (measured in dollars), $\mathbf{I}$ is average consumer income (measured in thousands of dollars), $\mathbf{P_E}$ is tomorrow's price of gasoline per gallon (measured in dollars).

a) Determine marginal value of $\mathbf{Q_G^D}$ with respect to each variable.

Solution:

Marginal value of $\mathbf{Q_G^D}$ with respect to $P_G = \frac{\partial \mathbf{Q_G^D}}{\partial P_G} = -2$. (holding other variables constant)

- It indicates that a $1 increase in the current price of gasoline (P_G) will cause a 2-million gallon decrease in the market demand for gasoline.

Marginal value of $\mathbf{Q_G^D}$ with respect to $I = \frac{\partial \mathbf{Q_G^D}}{\partial I} = +0.1$. (holding other variables constant)

- It indicates that a $1-thousand increase in average consumer income (I) will cause a 0.1-million gallon increase in the market demand of gasoline.

Marginal value of $\mathbf{Q_G^D}$ with respect to $P_E = \frac{\partial \mathbf{Q_G^D}}{\partial P_E} = +4$. (holding other variables constant)

- It indicates that a $1 increase in the future price of gasoline (P_E) will cause a 4-million gallon increase in the market demand of gasoline.

b) Derive the market demand equation, holding non-price determinants constant. Assume that I = 60 and $P_E = 1$.

Solution: "*Holding non-price determinants constant*" means substituting I and P_E into the market demand function.

$$Q_G^D = 10 - 2P_G + 0.1(60) + 4(1) = 10 - 2P_G + 6 + 4.$$

$$\mathbf{Q_G^D = 20 - 2P_G} \longrightarrow$$ The direct market demand equation

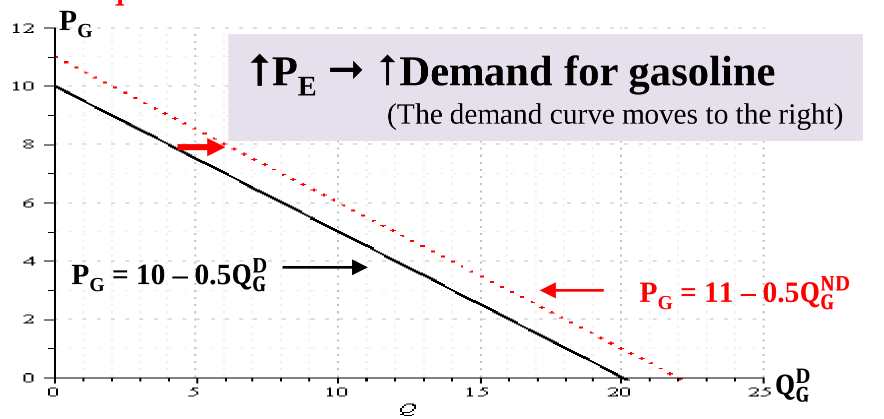
c) Draw the demand curve of gasoline.

Solution: **to draw the demand curve,**

we use the inverse demand equation, P = f(Q).

The inverse demand equation for gasoline

is $\mathbf{P_G = 10 - 0.5Q_G^D}$.



d) Suppose that a new report shows that the price of gasoline is going up to $1.50 at midnight. What is the new market demand equation after increase in the gas price?

Solution:

New direct market demand equation: $Q_G^{ND} = 10 - 2P_G + 0.1(60) + 4(1.5) = 10 - 2P_G + 6 + 6.$

$$\mathbf{Q_G^{ND} = 22 - 2P_G.}$$

New inverse market demand equation:

$$\mathbf{P_G = 11 - 0.5Q_G^{ND}}$$

Managerial Question

- From the time Apple launched iTunes in mid-2003 through early 2009, it charged 99 cents for each song.
- Music producers/singers wanted Apple to charge more.

Question: **Should iTunes increase or decrease the price?**

- **The question is quite important in the business world.**

$$\Delta P \rightarrow \Delta TR \rightarrow \Delta \pi,$$

$$\text{where } TR = PQ, \text{ and } \pi = TR - TC.$$

- It can be answered the question by the measurement of *elasticity*.
- ***Elasticity* is the most common measure of the sensitivity of the demand function to changes in any of its determinants.**

Measurement of Elasticity

Definition: **Elasticity** (denoted by η, the Greek letter "**eta**") is a measure of the sensitivity of one variable (**Y**) to changes in another variable (**X**). To calculate elasticity (η), there are 3 formulas as follows:

1. The specific equation is **not given**. The units of **Y** and **X** are given as a percentage.

$$\eta = \frac{\%\ \text{Change in Y}}{\%\ \text{Change in X}} = \frac{\%\Delta Y}{\%\Delta X}. \qquad (3)$$

2. The specific equation is **not given**. Two different points of **Y** and **X** are given, Y^1, Y^2, X^1, X^2.

$$\eta = \frac{\%\Delta Y}{\%\Delta X} = \frac{\frac{\Delta Y}{(Y^1 + Y^2)/2}}{\frac{\Delta X}{(X^1 + X^2)/2}} = \frac{\Delta Y}{\Delta X}\frac{(X^1 + X^2)}{(Y^1 + Y^2)} = \frac{(Y^2 - Y^1)}{(X^2 - X^1)}\frac{(X^1 + X^2)}{(Y^1 + Y^2)}. \qquad (4)$$

- Equation (4) is called as **midpoint elasticity or arc elasticity.**

3. The specific equation **is given**, **Y = f(X)**. One points of **Y** and **X** are given.

$$\eta = \frac{\text{Percentage change in the dependent variable}}{\text{Percentage change in the independent variable}} = \frac{\%\Delta Y}{\%\Delta X} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{\Delta Y}{\Delta X}\frac{X}{Y} = \frac{dY}{dX}\frac{X}{Y}. \qquad (5)$$

- Equation (5) is called as **point elasticity**.
- As the change in **X** becomes very small, $\Delta X \to 0$; the ratio $\Delta Y/\Delta X$ converges to the derivative of **Y** with respect to **X** (dY/dX).

The elasticity for **Y = f(X)** is defined as:

$$\eta = \frac{\%\ \textbf{Change in Y}}{\%\ \textbf{Change in X}} = \frac{\%\Delta Y}{\%\Delta X}.$$

The demand elasticities for $\mathbf{Q_X = f(P_X, P_Y, I, A)}$ is defined as:

Elasticities of Demand	**Y**		**Formula**
1. Own-price elasticity of demand (η_X)	Q_X	P_X	$\eta_X = \frac{\%\Delta Q_X}{\%\Delta P_X}$
2. Cross-price elasticity of demand (η_{XY})	Q_X	P_Y	$\eta_{XY} = \frac{\%\Delta Q_X}{\%\Delta P_Y}$
3. Income elasticity of demand (η_I)	Q_X	I	$\eta_I = \frac{\%\Delta Q_X}{\%\Delta I}$
4. Advertising elasticity of demand (η_A)	Q_X	A	$\eta_A = \frac{\%\Delta Q_X}{\%\Delta A}$

1. Own-Price Elasticity of Demand (η_X) (or Price Elasticity of Demand)

- The price elasticity of demand, η_X, measures the sensitivity of demand of good X to changes in the price of good X.

- By definition, η_X is defined to be the percent change in quantity demanded of good X (Q_X) divided to the percent change in price of good X (P_X).

$$\eta_X = \frac{\%\Delta Q_X}{\%\Delta P_X}. \tag{6}$$

- **The midpoint price elasticity of demand**, given $Q_X^1, Q_X^2, P_X^1, P_X^2$.

$$\eta_X = \frac{\%\Delta Q_X}{\%\Delta P_X} = \frac{\dfrac{\Delta Q_X}{(Q_x^1 + Q_x^2)/2}}{\dfrac{\Delta P_X}{(P_x^1 + P_x^2)/2}} = \frac{\Delta Q_X}{\Delta P_X}\frac{(P_x^1 + P_x^2)}{(Q_x^1 + Q_x^2)} = \frac{(Q_x^2 - Q_x^1)}{(P_x^2 - P_x^1)}\frac{(P_x^1 + P_x^2)}{(Q_x^1 + Q_x^2)} \tag{7}$$

- **The point price elasticity of demand**, given $Q_X = f(P_X)$.

$$\eta_X = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X}. \tag{8}$$

Example 4

The quantity of new cars increases by 10 percent. If the price elasticity of demand for new cars is -1.25, **the price of new cars** will fall by

Answer: 8 percent

Example 5

In the first week after Apple's iTunes raised the price on its most popular songs from 99 cents to $1.29, the quantity demanded of Doja Cat's "Say So" fell to 58,000 units from the 60,000 units sold in the previous week. **What is the price elasticity of demand for "Say So"?**

$$\eta_X = \frac{\%\Delta Q_X}{\%\Delta P_X} = \frac{(Q_X^2 - Q_X^1)}{(P_X^2 - P_X^1)} \frac{\left(P_X^1 + P_X^2\right)}{(Q_X^1 + Q_X^2)}$$

$$\eta_X = \frac{(58{,}000-60{,}000)}{(1.29-0.99)} \frac{(0.99+1.29)}{(60{,}000+58{,}000)} = \frac{-2000}{0.3} \frac{2.28}{118{,}000} = -0.13.$$

Example 6

The demand for a product is **Q = 9 – 0.7P + 2I**.
Assume that per capita income I is $13. When the price of a product is P = $30, **the price elasticity of demand** is

Answer: -1.5

Properties of Own-Price Elasticity of Demand

1. Own-price elasticity of demand is a negative number; $-\infty \leq \eta_X \leq 0$.

- $-\infty < \eta_X < -1$ indicates that the demand is **price elastic**, possibly because a good has many close substitutes.
  - A change in price causes a larger percentage change in quantity demanded.
  - The elastic demand is highly responsive, or sensitive, to changes in the price.
- $-1 < \eta_X < 0$ indicates that the demand is **price inelastic** because a good has few close substitutes.
  - A change in price causes a smaller percentage change in quantity demanded.
  - Gasoline is a good example because most people need it. When the prices of gasoline go up, consumers still buy it. The demand for gasoline does not change greatly.

Two extreme cases:

1. $\eta_X = 0$ indicates that a good is perfectly inelastic demand.

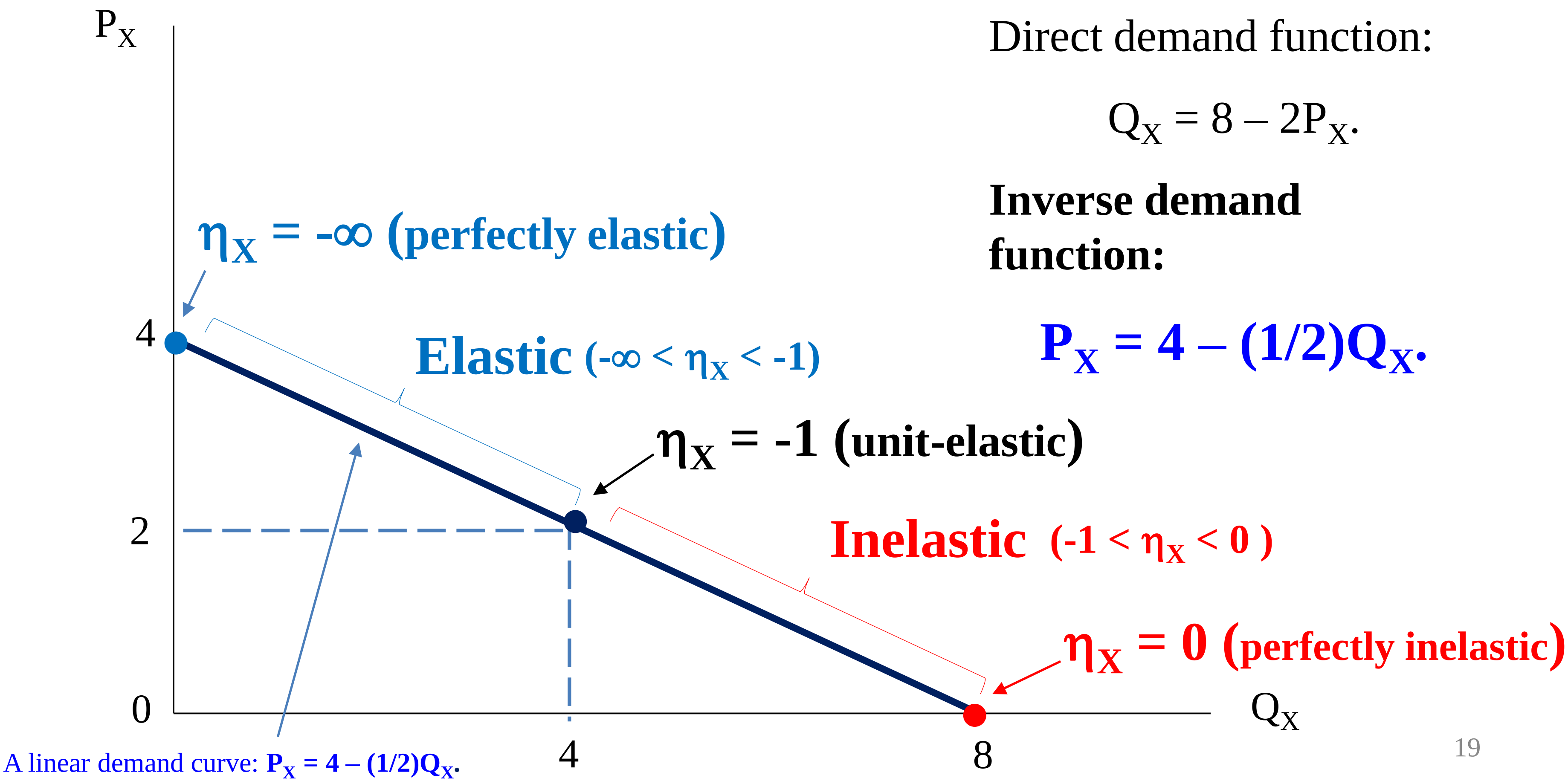
- A change in its price (P) will cause no change in the quantity demanded (Q).
- The quantity demanded is independent of price.
- Example: Lifesaving drugs.

2. $\eta_X = -\infty$ indicates that a good is perfectly elastic demand.

- The quantity demanded is very sensitive to price.
- Any increase in the price will cause demand to fall to zero.
- Example: any homogenous product with many close substitutes, e.g., coffee beans, potatoes.

Properties of Price Elasticity (cont.)

2. Given a linear demand curve, η_X is not constant along the curve. η_X increases in absolute value as price rises, approaching negative infinity as quantity approaches zero.



Direct demand function:

$$Q_X = 8 - 2P_X.$$

Inverse demand function:

$$P_X = 4 - (1/2)Q_X.$$

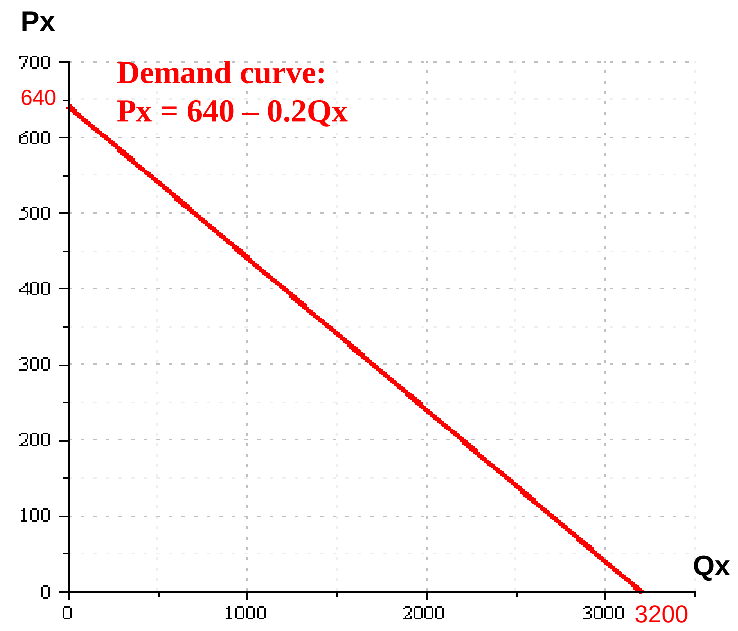
Example 7: Point elasticity

Given $$Qx = 1000 - 5Px + 0.1I + 10P_A - 2P_B$$

where Qx = Quantity demanded for inkjet printers; Price of a printer (Px) = \$80, Income (I) = \$20,000; Price of good A (P_A) = \$50; Price of good B (P_B) = \$150.

1. Derive the direct demand equation Qx = f(P_X), holding the non-price determinants constant.

$Qx = 1000 - 5Px + 0.1(\$20{,}000) + 10(\$50) - 2(\$150)$
$Qx = 1000 - 5Px + 2200$
$Qx = 3200 - 5Px$.



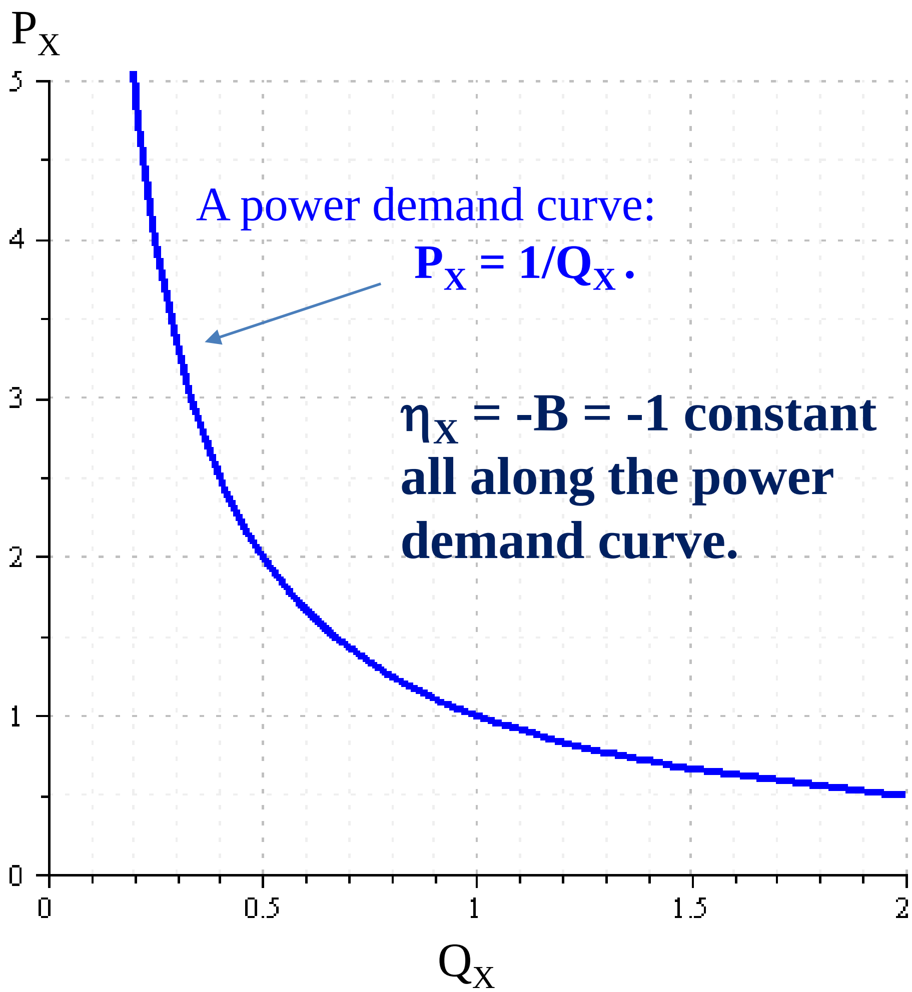
2. At what prices, if any, will the demand for computers be of unitary elasticity (i.e., $\eta_X = -1$)? **(P_X = \$320)**

3. What is the price elasticity of demand if price equals \$520? **(At P_X = \$520, $\eta_X = -4.3$)**

4. What is the price elasticity of demand if price equals \$80?
(At P_X = \$80, $\eta_X = -0.14$)

Properties of Price Elasticity (cont.)

3. Given a power demand function: $Q_X = AP_X^{-B}$, (9)
where A and B > 0. $\eta_X = -B$ everywhere on the demand curve.



So, the power demand function is called iso-elastic (constant elasticity).

For example, suppose that A = B = 1,

$$Q_X = \frac{1}{P_X} = P_X^{-1}. \quad (10)$$

$$\eta_X = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X} = (-1)P_X^{-2}\frac{P_X}{P_X^{-1}}.$$

$$\eta_X = -1.$$

Note that: A power demand function is also called a Cobb-Douglas demand function.

A Cobb-Douglas function (for 2 independent variables) takes the form of

$Y = AX_1^aX_2^b$, where A, a, and b are constant.

Example 8
Power & Linear Demand Functions

1. Power demand equation: $\mathbf{Q_X = 2P_X^{-4}}$.

$$\eta_X = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X} = (-4)2P_X^{-5}\frac{P_X}{2P_X^{-4}} = -4.$$

- So, η_X of the power demand equation is constant.

2. Linear demand equation: $\mathbf{Q_X = 2 - 4P_X}$.

$$\eta_X = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X} = (-4)\frac{P_X}{Q_X}.$$

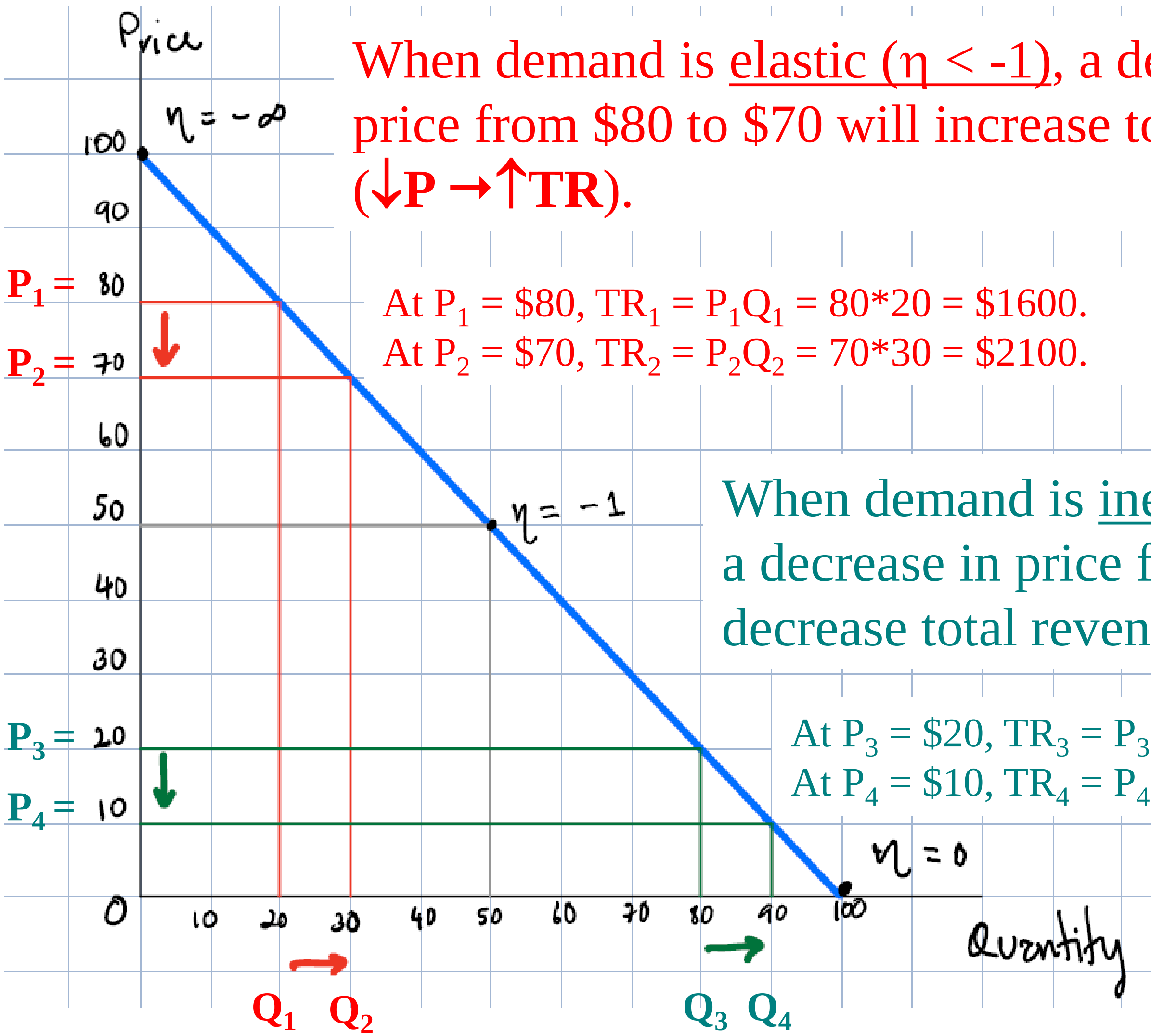
- So, η_X of the linear demand equation depends on $\frac{P_X}{Q_X}$.
- The different values of P_X and Q_X will have the different values of η_X.

Effect of Price Elasticity on the Firm’s Total Revenue

Total Revenue and Price Elasticity

- Total Revenue: **$TR = P_X \times Q_X$.** (11)
- When managers change the price of a product, TR also changes.
- Managers need to decide whether a price change will increase the firm's total revenue. The decision depends on the price elasticity of demand.

Demand, Price Elasticity, and Total Revenue



When demand is elastic ($\eta < -1$), a decrease in price from \$80 to \$70 will increase total revenue (↓**P** →↑**TR**).

At P_1 = \$80, $TR_1 = P_1Q_1$ = 80*20 = \$1600.
At P_2 = \$70, $TR_2 = P_2Q_2$ = 70*30 = \$2100.

When demand is inelastic ($-1 < \eta < 0$), a decrease in price from \$20 to \$10 will decrease total revenue (↓**P** →↓**TR**).

At P_3 = \$20, $TR_3 = P_3Q_3$ = 20*80 = \$1600.
At P_4 = \$10, $TR_4 = P_4Q_4$ = 10*90 = \$900.

Relationship between Price Elasticity (η_X) and Total Revenue (TR)

- **When demand is elastic** (η_X **< -1**), price and total revenue move in opposite direction.
  - **↑Px → ↓TR.**
  - **↓Px → ↑TR.**

A negative relationship between P_X and TR.

- **When demand is inelastic (-1** < η_X **< 0),** price and total revenue move in the same direction.
  - **↑Px → ↑TR.**
  - **↓Px → ↓ TR.**

A positive relationship between P_X and TR.

Note that calculus proof of the relationship between TR and η_X is in Math Review, Slide#53, Appendix A.

Price Elasticity, Total Revenue, and Marginal Revenue

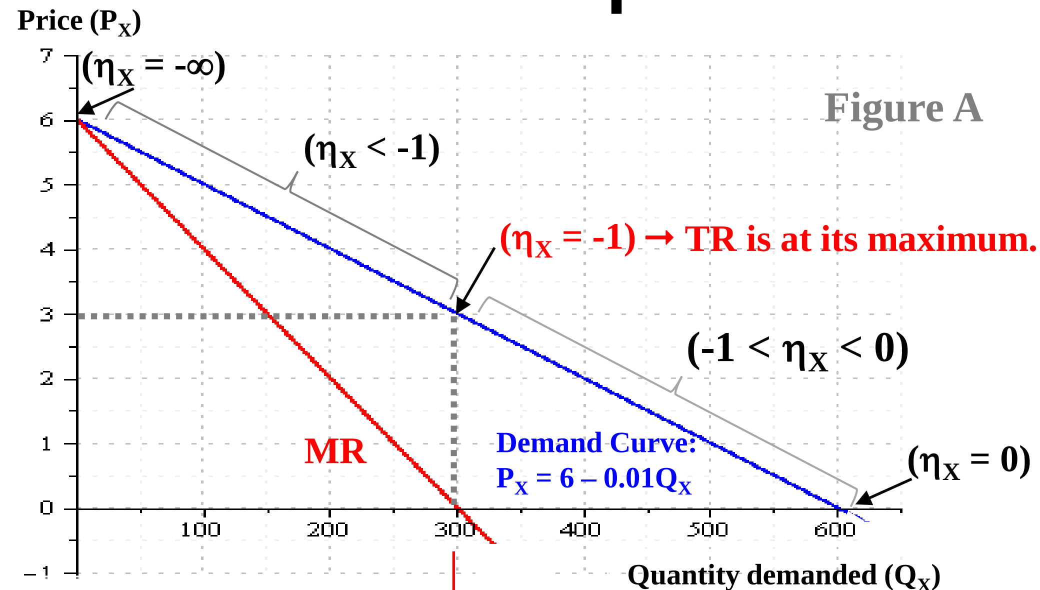
- TR = the total amount of money consumers spend on the product in a given time period.
- Marginal Revenue (MR): MR is the change in total revenue for a unit change in quantity demanded.
- **$MR = dTR/dQ_X$** (derivative of TR with respect to Q_X).
- MR = Slope of total revenue curve.

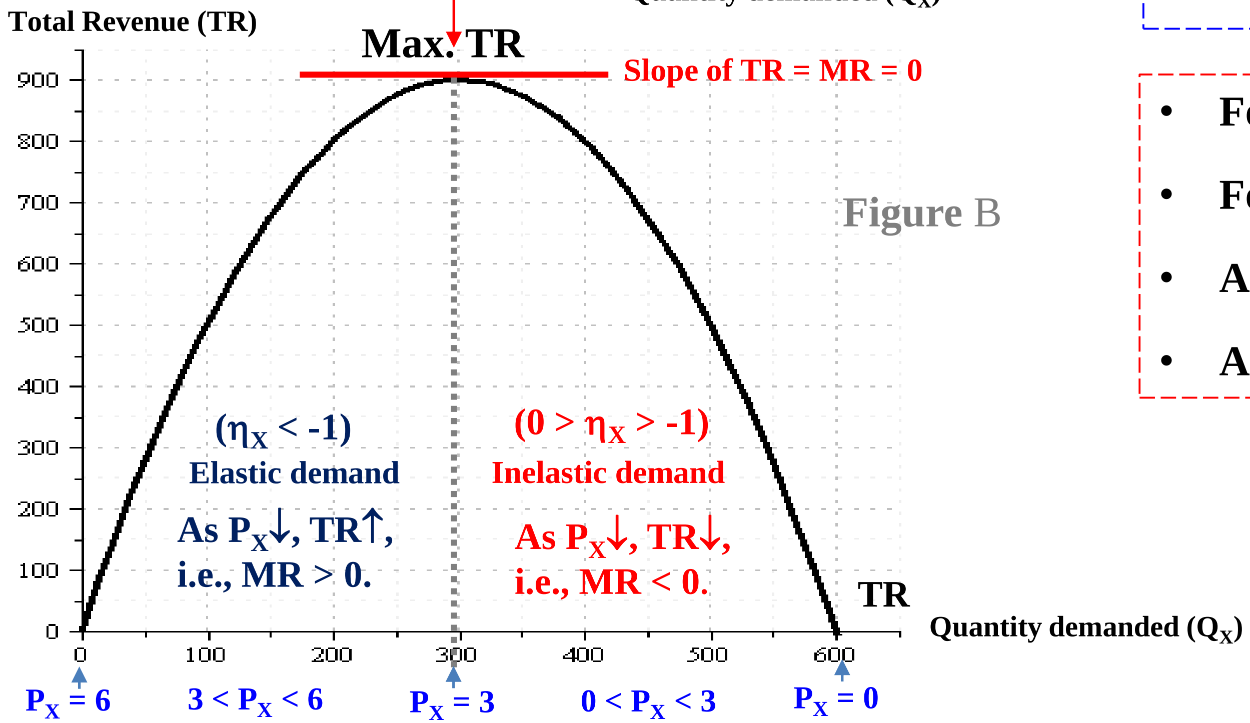
Example 9: A direct demand equation: $Q_X = 600 – 100P_X$

An inverse demand equation: $P_X = 6 – 0.01Q_X$

- **$TR = P_X Q_X = (6 – 0.01Q_X)Q_X = 6Q_X – 0.01Q_X^2$.**
- **$MR = dTR/dQ_X = 6 – 0.02Q_X$.**

Relationship between P_X, η_X, MR, and TR





The direct demand equation:

$$Q_X = 600 – 100P_X$$

The inverse demand equation:

$$P_X = 6 – 0.01Q_X$$

The total revenue equation:

$$TR = 6Q_X – 0.01Q_X^2$$

The marginal revenue equation:

$$MR = 6 – 0.02Q_X$$

- For $3 < P_X < 6$, $\eta_X < -1$.
- For $0 < P_X < 3$, $-1 < \eta_X < 0$.
- At $P_X = 6$, $\eta_X = -\infty$.
- At $P_X = 0$, $\eta_X = 0$.

Marginal Revenue and Price Elasticity

- MR varies along a demand curve for different quantities and prices.
- Since MR depends on the slope of the demand curve and so does elasticity, it turns out that one can write MR for any point on the demand curve as a function of the price and the price elasticity at that point on the demand curve.

$$\mathbf{MR = P_X\left(1 + \frac{1}{\eta_X}\right)} \qquad (12)$$

- **Example 10**: If price is \$25 when the price elasticity of demand is −0.5, then marginal revenue must be:

$$\text{MR} = \$25\left(1 + \frac{1}{(-0.5)}\right) = -25.$$

Note that calculus proof of the relationship between MR and η_X is in Math Review, Slide#54, Appendix A.

Profit-maximizing Price and Price Elasticity

- The profit maximization condition (see marginal analysis in Math Review):

$$\mathbf{MR = MC}.$$

- Using Equation (12), the profit-maximizing condition is

$$\mathbf{P_X^*\left(1+\frac{1}{\eta_X}\right) = MC}.$$

- The profit-maximizing price is

$$\mathbf{P_X^* = \frac{MC}{\left(1+\frac{1}{\eta_X}\right)}}. \qquad (13)$$

- **Example 11**: A monopoly incurs a marginal cost of \$1 for each unit produced. If the price elasticity of demand equals -2.0, calculate the profit-maximizing price ($\mathrm{P_X^*}$).

$$P_X^* = \frac{1}{\left(1+\frac{1}{(-2)}\right)} = \$2.$$

2. Cross-price Elasticity of Demand (η_{XY})

- The cross-price elasticity of demand, η_{XY}, measures the sensitivity of demand to changes in the prices of the other product.

- By definition, η_{XY} is defined to be the percent change in quantity demanded of good X (Q_X) divided to the percent change in a price of good Y (P_Y).

$$\eta_{XY} = \frac{\%\Delta Q_X}{\%\Delta P_Y}. \qquad (14)$$

- **The midpoint cross-price elasticity of demand**, given $Q_X^1, Q_X^2, P_Y^1, P_Y^2$.

$$\eta_{XY} = \frac{\dfrac{\Delta Q_X}{(Q_X^1 + Q_X^2)/2}}{\dfrac{\Delta P_Y}{(P_Y^1 + P_Y^2)/2}} = \frac{(Q_X^2 - Q_X^1)}{(P_Y^2 - P_Y^1)} \frac{(P_Y^1 + P_Y^2)}{(Q_X^1 + Q_X^2)}. \qquad (15)$$

- **The point cross-price elasticity of demand**, given $Q_X = f(P_X, P_Y)$.

$$\eta_{XY} = \frac{\partial Q_X}{\partial P_Y} \frac{P_Y}{Q_X}. \qquad (16)$$

Cross-price elasticity of demand

$$\eta_{XY} = \frac{\partial Q_X}{\partial P_Y}\frac{P_Y}{Q_X} \qquad (16)$$

Sign of η_{XY} depends on $\frac{\partial Q_x}{\partial P_y}$.

- **If $\frac{\partial Q_x}{\partial P_y} > 0$, $\eta_{XY} > 0$.** That means the two products are substitutes.

  Example: Wheat and corn, tea and coffee, Xbox and Nintendo Switch

- **If $\frac{\partial Q_x}{\partial P_y} < 0$, $\eta_{XY} < 0$.** That means the two products are complements.

  Example: Computers and computer software, tablets and applications

- **If $\frac{\partial Q_x}{\partial P_y} = 0$, $\eta_{XY} = 0$.** That means the two products are independent.

  Example: Butter and airline ticket.

Example 12:
Cross-price point elasticity of demand

- Given: $\mathbf{Q_X = 1000 - 0.2P_X + 0.5P_Y + 0.04I}$,
- Let $Q_X = 100$ units and $P_Y = \$20$.
- Calculate cross-price elasticity (η_{XY})

$$\eta_{XY} = \frac{\partial Q_X}{\partial P_Y}\frac{P_Y}{Q_X} = 0.5\frac{20}{100} = 0.1.$$

- Since η_{XY} is positive, Goods X and Y are substitutes.

3. Income Elasticity of Demand (η_I)

- The income elasticity of demand, η_I, measures the sensitivity of demand to changes in buyers' incomes.

- By definition, η_I is defined to be the percent change in quantity demanded of good X (Q_X) divided to the percent change in income (I).

$$\eta_I = \frac{\%\Delta Q_X}{\%\Delta I}. \qquad (17)$$

- **The midpoint income elasticity of demand**, given Q_X^1, Q_X^2, I^1, I^2.

$$\eta_I = \frac{\frac{\Delta Q_X}{(Q_X^1 + Q_X^2)/2}}{\frac{\Delta I}{(I^1 + I^2)/2}} = \frac{(Q_X^2 - Q_X^1)}{(I^2 - I^1)} \frac{(I^1 + I^2)}{(Q_X^1 + Q_X^2)}. \qquad (18)$$

- **The point income elasticity of demand**, given $Q_X = f(P_X, I)$.

$$\eta_I = \frac{\partial Q_X}{\partial I} \frac{I}{Q_X}. \qquad (19)$$

- Note that income can be defined as aggregate consumer income or as per capita income.

Income Elasticity of Demand

$$\eta_I = \frac{\%\Delta Q_X}{\%\Delta I} = \frac{\partial Q_X}{\partial I}\frac{I}{Q_X} \qquad (19)$$

- Income elasticity of demand depends on **the sign of** $\frac{\partial Q_X}{\partial I}$.
  - **If** $\frac{\partial Q_X}{\partial I} > 0$, $\eta_I > 0$.
    - **A normal good** is one for which an increase in income leads to an increase in demand.
  - **If** $\frac{\partial Q_X}{\partial I} < 0$, $\eta_I < 0$.
    - **An inferior good** is one for which an increase in income leads to a decrease in demand.

Example 13: Income elasticity of demand

- Given: $\mathbf{Q_X = 1000 - 0.2P_X + 0.5P_Y + 0.04I}$,
- Let $Q_X = 2000$, $P_Y = 500$, and $I = 10000$.
- Calculate income elasticity

$$\eta_I = \frac{\partial Q_X}{\partial I}\frac{I}{Q_X} = 0.04\frac{10000}{2000} = 0.2.$$

- The income elasticity for good X is 0.2, which means that a 1% increase in income cause demand for Good X to increase by 0.2%.
- Since η_I is positive, Good X is normal.

4. Advertising Elasticity of Demand (η_A)

- The advertising elasticity of demand, η_A, measures the sensitivity of demand to change in the sellers' advertising expenditure.
- By definition, η_A is defined to be the percent change in quantity demanded of good X (Q_X) divided to the percent change in advertising expenditure (A).

$$\eta_A = \frac{\%\Delta Q_X}{\%\Delta A}. \qquad (20)$$

- **The midpoint advertising elasticity of demand**, given Q_X^1, Q_X^2, A^1, A^2.

$$\eta_A = \frac{\dfrac{\Delta Q_X}{(Q_X^1 + Q_X^2)/2}}{\dfrac{\Delta A}{(A^1 + A^2)/2}} = \frac{(Q_X^2 - Q_X^1)}{(A^2 - A^1)} \frac{(A^1 + A^2)}{(Q_X^1 + Q_X^2)} \qquad (21)$$

- **The point advertising elasticity of demand**, $Q_X = f(P_X, A)$.

$$\eta_A = \frac{\%\Delta Q_X}{\%\Delta A} = \frac{\partial Q_X}{\partial A} \frac{A}{Q_X}. \qquad (22)$$

Example 14: Advertising Elasticity of Demand

Given: $\mathbf{Q_X = 500 - 0.5P_X + 0.01I + 0.02A}$
and advertising-to-sales ratio $(A/Q_X) = 2$.

Calculate advertising elasticity of demand.

$$\eta_A = \frac{\partial Q_X}{\partial A}\frac{A}{Q_X} = 0.02(2) = 0.04.$$

- The advertising elasticity for good X is 0.04, which means that a 1% increase in advertising expenditure will increase the demand for good X by 0.04%.

Recap

- Market demand function
- Elasticities: η_X, η_{XY}, η_I, and η_A
  - Midpoint and point elasticities
- The own-price elasticity:
  - Linear demand function
  - Power demand function
  - The relationship between TR, MR, and η_X.
  - η_X and optimal pricing policy

$$MC = P_X\left(1 + \frac{1}{\eta_X}\right) \longrightarrow P_X^* = \frac{MC}{\left(1 + \frac{1}{\eta_X}\right)}$$

Extra Questions

1. Suppose the market demand curve for pizza can be expressed as **$Q_D = 100 - 2P + 3Pb$**, where Q_D is the quantity of pizza demanded, P is the price of pizza, and Pb is the price of a burrito. **What is the relationship between burritos and pizza, from the point of view of consumers?**

   A. They are independent.
   B. They are complements.
   C. They are substitutes.
   D. They are inferior goods.
   E. Not enough information to answer the question.

Answer: C

2. When the price of bananas is 50 cents a pound, the total demand is 100 pounds. If the price increases to 60 cents, the total demand drops to 60 pounds. **What is the price elasticity of demand?**

Answer: -2.75

3. When the price of bananas increases from 50 cents to 60 cents, the total demand drops 50%. **What is the price elasticity of demand?**

Answer: -2.75

4. A fall in the price of X from $12 to $8 causes an increase in the quantity of Y demanded from 900 to 1100 units. **What is the cross-price elasticity of demand?**

Answer: η_{YX} = -0.5

5. If the demand for coffee is **Q = 25000 – 0.5P,** where Q is the number of tons produced and P is the price per ton, total revenue is maximized when the output level (Q*) is

Answer: Q* = 12500 tons.

6. The demand for tickets to a concert is given by

$Q = 1000P^{-1.5}I^{2}$ where P is the price of a ticket. Assume that per capita income, I, is $2000. At a price, P, of $70 the price elasticity of demand is

Answer: $\eta = -1.5$.

7. Demand for Post Malone CDs is equal to $\mathbf{Q_M = P_M^{-2} I^{1.5} P_E^{4}}$ where Q_M is the quantity of Post Malone CDs, P_M is the price of a Malone CD, I is per capital income, and P_E is the price of a Billie Eilish CD.

a. Calculate the cross-price elasticity (η_{ME}).

b. What is the relationship between Malone and Eilish CDs?

Answer: a) $\eta_{ME} = 4$ b) substitutes